2nd IMO 1960

A1. Determine all 3 digit numbers N which are divisible by 11 and where N/11 is equal to the sum of the squares of the digits of N.

A2. For what real values of x does the following inequality hold:

$4x2/(1 - v(1 + 2x))2 < 2x + 9$?

A3. In a given right triangle ABC, the hypoteneuse BC, length a, is divided into n equal parts with n an odd integer. The central part subtends an angle a at A. h is the perpendicular distance from A to BC. Prove that:

$\tan a = 4nh/(an2 - a)$.

B1. Construct a triangle ABC given the lengths of the altitudes from A and B and the length of the median from A.

B2. The cube ABCDA'B'C'D' has A above A', B above B' and so on. X is any point of the face diagonal AC and Y is any point of B'D'.

(a) find the locus of the midpoint of XY;

(b) find the locus of the point Z which lies one-third of the way along XY, so that ZY=2·XZ.

B3. A cone of revolution has an inscribed sphere tangent to the base of the cone (and to the sloping surface of the cone). A cylinder is circumscribed about the sphere so that its base lies in the base of the cone. The volume of the cone is V1 and the volume of the cylinder is V2.

(a) Prove that V1 ? V2;

(b) Find the smallest possible value of V1/V2. For this case construct the half angle of the cone.

B4. In the isosceles trapezoid ABCD (AB parallel to DC, and BC = AD), let AB = a, CD = c and let the perpendicular distance from A to CD be h. Show how to construct all points X on the axis of symmetry such that ?BXC = ?AXD = 90o. Find the distance of each such X from AB and from CD. What is the condition for such points to exist?