

5th IMO 1963

A1. For which real values of p does the equation

$\sqrt{x^2 - p} + 2\sqrt{x^2 - 1} = x$ have real roots? What are the roots?

A2. Given a point A and a segment BC , determine the locus of all points P in space for which $\angle APX = 90^\circ$ for some X on the segment BC .

A3. An n -gon has all angles equal and the lengths of consecutive sides satisfy $a_1 = a_2 = \dots = a_n$. Prove that all the sides are equal.

B1. Find all solutions x_1, \dots, x_5 to the five equations $x_i + x_{i+2} = y x_{i+1}$ for $i = 1, \dots, 5$, where subscripts are reduced by 5 if necessary.

B2. Prove that $\cos p/7 - \cos 2p/7 + \cos 3p/7 = 1/2$.

B3. Five students A, B, C, D, E were placed 1 to 5 in a contest with no ties. One prediction was that the result would be the order A, B, C, D, E . But no student finished in the position predicted and no two students predicted to finish consecutively did so. For example, the outcome for C and D was not 1, 2 (respectively), or 2, 3, or 3, 4 or 4, 5. Another prediction was the order D, A, E, C, B . Exactly two students finished in the places predicted and two disjoint pairs predicted to finish consecutively did so. Determine the outcome.