

## 12th IMO 1970

A1.  $M$  is any point on the side  $AB$  of the triangle  $ABC$ .  $r$ ,  $r_1$ ,  $r_2$  are the radii of the circles inscribed in  $ABC$ ,  $AMC$ ,  $BMC$ .  $q$  is the radius of the circle on the opposite side of  $AB$  to  $C$ , touching the three sides of  $AB$  and the extensions of  $CA$  and  $CB$ . Similarly,  $q_1$  and  $q_2$ . Prove that  $r_1 r_2 q = r q_1 q_2$ .

A2. We have  $0 < x_i < b$  for  $i = 0, 1, \dots, n$  and  $x_n > 0$ ,  $x_{n-1} > 0$ . If  $a > b$ , and  $x_n x_{n-1} \dots x_0$  represents the number  $A$  base  $a$  and  $B$  base  $b$ , whilst  $x_n - 1 x_{n-2} \dots x_0$  represents the number  $A'$  base  $a$  and  $B'$  base  $b$ , prove that  $A'B < AB'$ .

A3. The real numbers  $a_0, a_1, a_2, \dots$  satisfy  $1 = a_0 = a_1 = a_2 = \dots$ .  $b_1, b_2, b_3, \dots$  are defined by  $b_n = \prod_{k=1}^n (1 - a_{k-1}/a_k)$ .

(a) Prove that  $0 < b_n < 2$ .

(b) Given  $c$  satisfying  $0 < c < 2$ , prove that we can find an  $n$  so that  $b_n > c$  for all sufficiently large  $n$ .

B1. Find all positive integers  $n$  such that the set  $\{n, n+1, n+2, n+3, n+4, n+5\}$  can be partitioned into two subsets so that the product of the numbers in each subset is equal.

B2. In the tetrahedron  $ABCD$ , angle  $BDC = 90^\circ$  and the foot of the perpendicular from  $D$  to  $ABC$  is the intersection of the altitudes of  $ABC$ . Prove that:

$$(AB + BC + CA)^2 = 6(AD^2 + BD^2 + CD^2).$$

When do we have equality?

B3. Given 100 coplanar points, no 3 collinear, prove that at most 70% of the triangles formed by the points have all angles acute.