20th IMO 1978

A1. m and n are positive integers with $m < n$. The last three decimal digits of 1978^m are the same as the last three decimal digits of 1978^n. Find m and n such that $m + n$ has the least possible value.

A2. P is a point inside a sphere. Three mutually perpendicular rays from P intersect the sphere at points U, V and W. Q denotes the vertex diagonally opposite P in the parallelepiped determined by PU, PV, PW. Find the locus of Q for all possible sets of such rays from P.

A3. The set of all positive integers is the union of two disjoint subsets $\{f(1), f(2), f(3), ... \}$, $\{g(1), g(2), g(3), ... \}$, where $f(1) < f(2) < f(3) < ...$, and $g(1) < g(2) < g(3) < ...$, and $g(n) = f(f(n)) + 1$ for $n = 1, 2, 3, ...$. Determine $f(240)$.

B1. In the triangle ABC, AB = AC. A circle is tangent internally to the circumcircle of the triangle and also to AB, AC at P, Q respectively. Prove that the midpoint of PQ is the center of the incircle of the triangle.

B2. {ak} is a sequence of distinct positive integers. Prove that for all positive integers n, ?1=k=n ak/k2 = ?1=k=n 1/k.

B3. An international society has its members from six different countries. The list of members has 1978 names, numbered 1, 2, ... , 1978. Prove that there is at least one member whose number is the sum of the numbers of two members from his own country, or twice the number of a member from his own country.